STATISTICS 2 (A) TEST PAPER 1 : ANSWERS AND MARK SCHEME

1. (a) Census considers whole population; survey looks at a subset — B1
   (b) Census : ask the whole village Survey : ask a proportion — B1 B1
   (c) The sampling units are the individual residents — B1 — 4

2. (a) $X \sim \text{Po}(1{\cdot}5)$ From tables, $P(X = 0) = 0{\cdot}223$ — B1
   (b) $P(X > 5) = 1 - 0{\cdot}9955 = 0{\cdot}0045$ — M1 A1
   (c) $P(X \leq 3) = 0{\cdot}9344$ and $P(X \leq 4) = 0{\cdot}9814$, so he needs 4 copies — M1 M1 A1 — 6

3. (a) No. of '6's $\sim B(60, p) \approx \text{Po}(60p)$. $H_0 : p = \frac{1}{6}$ $H_1 : p \neq \frac{1}{6}$ — B1 B1
   Under H_0, $P(X \geq 16 \text{ or } X \leq 4) = 0{\cdot}0487 + 0{\cdot}0293 = 0{\cdot}078 > 5\%$ — M1 A1 A1
   Do not reject H_0 at 5% significance level; accept that $p = \frac{1}{6}$ — A1
   (b) Now $H_1 : p > \frac{1}{6}$ $P(X \geq 16) = 0{\cdot}0487 < 5\%$, so reject H_0 — B1 M1 A1 A1 — 10

4. (a) Need $F(x) = 0{\cdot}5$, so $x^2 = 58$ $x = \sqrt{58} = 7{\cdot}62$ — M1 A1
   (b) $\frac{1}{84}(p^2 - 16) = 0{\cdot}25 : p = 6{\cdot}083$ $\frac{1}{84}(q^2 - 16) = 0{\cdot}75 : q = 8{\cdot}888$ — M1 A1 A1
   IQR $= 8{\cdot}888 - 6{\cdot}083 = 2{\cdot}81$ — A1
   (c) $f(x) = F'(x) = \frac{x}{42}$, $4 \leq x \leq 10$; $f(x) = 0$ otherwise — M1 A1 A1
   (d) Graph drawn Mode = 10 : maximum value of $f(x)$ on graph — B1 M1 A1 — 12

5. (a) $R \sim B(15, 0{\cdot}4) : P(R < 2) = P(R \leq 1) = 0{\cdot}0052$ — B1 M1 A1
   (b) $P(R \geq 8) = 1 - P(R \leq 7) = 1 - 0{\cdot}7869 = 0{\cdot}213$ — M1 A1
   (c) Number of greens is $G \sim B(150, 0{\cdot}6) \approx N(90, 36)$ — M1 A1
   $P(G > 100{\cdot}5) = P(Z > 10{\cdot}5 \div 6) = P(Z > 1{\cdot}75) = 0{\cdot}0401$ — M1 A1 M1 A1 A1 — 12

6. (a) Mean $= \int_0^4 \frac{3}{64} t^3(4 - t)\, dt = \frac{3}{64}[t^4 - t^5/5]_0^4 = 2{\cdot}4$ — M1 A1
   $\text{Var}(T) = \int_0^4 \frac{3}{64} t^4(4 - t)\, dt - 2{\cdot}4^2 = \frac{3}{64}[4t^5/5 - t^6/6]_0^4 - 5{\cdot}76$ — M1 A1 A1
   $= 0{\cdot}64$ Standard deviation $= \sqrt{0{\cdot}64} = 0{\cdot}8$ — M1 A1
   (b) $P(T \leq 3) = \int_0^3 \frac{3}{64} t^2(4 - t)\, dt = 0{\cdot}738$ $P(T > 3) = 0{\cdot}262$ — M1 A1 M1 A1
   (c) $0{\cdot}738^2 = 0{\cdot}545$ (d) Unlikely that all recover within 4 days — M1 A1; B1 — 14

7. (a) (i) $e^{-0{\cdot}8} = 0{\cdot}449$ (ii) $0{\cdot}8e^{-0{\cdot}8} = 0{\cdot}359$ — B1 B1
   (b) $P(0) + P(1) = 0{\cdot}449^{10} + 10 \times 0{\cdot}449^9 \times 0{\cdot}359 = 0{\cdot}002996$ — M1 M1 A1 A1
   (c) No. in 10 patches $\sim$ Po(8); then $P(X < 2) = P(X \leq 1) = 0{\cdot}0030$ — B1 M1 A1
   (d) Good agreement, but Poisson is easier to calculate — B1 B1
   (e) In 1 m^2, expect 8000 daisies, so use $\text{Po}(8000) \approx N(8000, 8000)$ — M1 A1
   $P(X > 8100{\cdot}5) = P(Z > 100{\cdot}5/89{\cdot}44) = P(Z > 1{\cdot}12) = 0{\cdot}131$ — M1 A1 M1 A1 — 17